How Stress Variance in the Automotive Environment will Affect a ‘True’ Value of the Reliability Demonstrated by Accelerated Testing

Andre Kleyner
Delphi Automotive

ABSTRACT
This paper discusses the effect of the field stress variance on the value of demonstrated reliability in the automotive testing. In many cases the acceleration factor for a reliability demonstration test is calculated based on a high percentile automotive stress level, typically corresponding to severe user or environmental conditions. In those cases the actual field (‘true’) reliability for the population will be higher than that demonstrated by a validation test.

This paper presents an analytical approach to estimating ‘true’ field reliability based on the acceleration model and stress variable distribution over the vehicle population. The method is illustrated by an example of automotive electronics reliability demonstration testing.

KEYWORDS: accelerated testing, reliability demonstration, success based testing, sample size, stress variable, test to field correlation, user severity, population reliability, Weibull distribution


INTRODUCTION
Product validation and reliability demonstration by test are important parts of product development and are commonly included in automotive technical documentation specifying certain levels of reliability needed to be demonstrated by a product testing. Most automotive product specifications contain reliability requirements expressed as some form of reliability metrics, such as reliability $R(t)$, $B_{10}$ - life, PPM, MTBF or others. Demonstrating this metrics is often part of a product validation testing, either design validation (DV) or process validation (PV).

The two most common ways of demonstrating reliability are: success based testing and test-to-failure (see Appendix). However, the main limitation of the reliability demonstration testing (both success-based and test-to-failure) is in the required sample size (see Appendix). Therefore the reliability numbers demonstrated by either test are relatively low compared to what is typically required by product specifications on field reliability (often 99% and higher).

This problem often causes disputes between automotive OEMs and their suppliers and also generates additional, often unnecessary amount of testing. This issue however can be addressed by taking into account variation in the field stress and/or usage conditions (e.g. driving mileage, vibration level, temperature exposure, number of on/off cycles, etc.) If a test simulates more severe conditions than the average, which is the way it is typically done in the automotive industry, then the actual field reliability will be higher than that demonstrated by a test.

General Motors in its 2004 version of the worldwide engineering standard GMW3172 [1] provided a field to test reliability conversion table based on Customer Variability Ratio (CVR) = 99.8%-ile/50%-ile Damage. This table was helpful in comparing the reliability demonstrated during the test to the expected field reliability, however it was not specific to product failure modes and acceleration factors and was removed from the later editions of GMW 3172.

However the need for mathematical models to correlate the demonstrated reliability to the ‘true’ field reliability has remained, which prompted the work presented in this paper.

ACCELERATED TESTING AND FIELD STRESS VARIATION
Field stress clearly has a strong effect on the reliability of parts subject to that stress. Since different units in a vehicle population will be subjected to different stress levels, their individual reliability functions will vary, therefore the reliability estimate should be treated as population reliability. The
The concept of population reliability vs. individual reliability has been discussed previously by Lu and Anderson-Cook in [2] and [3].

Clearly, the higher the stress level the sooner you would expect a device to fail. Davis [4] formulates the reliability as a mathematical function affected by a wide variety of possible noise factors (e.g. part to part variation, wear-out, duty cycles, etc.). In a similar fashion, taking into account the effect of stress variables, mentioned in Introduction, the reliability of a field population can be expressed as

\[
R(t) = \int_{t \in S_n} f_{S_n}(t | s) f_n(s)ds dt
\]

where \( S_n \) = stress space with variables \( s_1, \ldots, s_n \)

\[
\int_{t \in S_n} = \text{a multiple integral over the entire stress space } S_n
\]

\( R_{S_n}(t | s) = \int_{t \in S_n} f_{S_n}(t | s)ds \) denotes the reliability at time \( t \) conditional on a given point in the stress space.

The effect of stress on reliability can be modeled using the concept of acceleration factor. The acceleration factor (AF) is defined as a ratio of the expected life in un-accelerated (field) conditions to the expected life under accelerated (test) conditions (see for example Nelson [5] or O'Connor and Kleyner [6]):

\[
AF = \frac{L_{\text{Field}}}{L_{\text{Test}}}
\]

where \( L_{\text{Field}} \) and \( L_{\text{Test}} \) are the expected product lives in field and test conditions respectively.

Acceleration factor relationship with reliability function is often modeled as [6]

\[
R_{\text{Field}}(t) = R_{\text{Test}} \left( \frac{t}{AF} \right)
\]

where:

\( R_{\text{Field}} \) = Reliability under field conditions

\( R_{\text{Test}} \) = Reliability under test conditions.

\( t \) = time in the field

Accelerated testing is usually done based on either an increased usage rate (e.g. increased number of loading cycles, extended number of driven miles, higher duty cycle, etc.) or based on an increased stress level (e.g., elevated temperature, vibration, voltage, etc.). In either case, if the total product population has variation in a usage rate or a stress level, the acceleration factor will also vary. This paper will focus on the cases where tests are accelerated by increased stress levels, although the approach to the increased usage rate analysis is very similar.

The acceleration factor \( AF \) is a function of multiple stress variables comprising the stress space \( S_n \), therefore \( AF = AF(s_1, \ldots, s_n) \), and thus would vary for each unit in the population depending on the stresses experienced by that unit. Therefore, by combining (1) and (3) the overall field reliability for the population can be re-expressed as

\[
R_{\text{Field}}(t)
= \int_{s_1}^{s_{max}} \ldots \int_{s_n}^{s_{max}} R_{\text{Test}} \left( \frac{t}{AF(s_1, \ldots, s_n)} \right) f_1(s_1) \ldots f_n(s_n) ds_1 \ldots ds_n.
\]

where:

\( s_{max} \) = maximum possible stress value for the variable \( s_i \) achieved in a product population

\( f(s) \) = probability density function (pdf) of the \( s \) distribution in a vehicle population

Equation (4) assumes that an accelerated test is designed properly, i.e. it would precipitate failure modes and failure mechanisms consistent with those experienced by the product in the field. Also this type of consistency often manifests itself in similar failure distributions. For example, both field and test failures would often follow Weibull distribution with similar shape parameters \( \beta \), but different scale parameters \( \eta \) (see Appendix).

Equation (4) is multivariate, however in many practical applications one dominant stress factor is often identified and applied to calculate the test acceleration factor. For example, for a temperature cycling test the temperature excursion \( \Delta T \) is often used as the key stress variable, dynamic acceleration typically dominates vibration testing, maximum ambient temperature usually defines a high temperature endurance test.
and so on. In the cases where the dominant stress variable can
be identified see Frenkel at. al. Chapter 27 [7], the field
reliability \( R_{\text{field}} \) can be reduced to a simplified equation:

\[
R_{\text{field}}(t) = \int_0^{\infty} R_{\text{ref}} \left( \frac{t}{AF(S)} \right) f(S) dS
\]

(5)

where \( S \) is the dominant stress variable.

If both field stress levels and product failures can be modeled
as random variables with probability distributions, equation (5)
can be re-written in a parametric form as

\[
R_{\text{field}}(t) = \int_0^{\infty} R_{\text{ref}} \left( \frac{t; \theta_R}{AF(S)} \right) f(S; \psi_S) dS
\]

(6)

where:

\[ \theta_R = \{ \theta_1, \ldots, \theta_q \} = \text{vector of failure (reliability) distribution parameters (dimension } q \} \]

\[ \psi_S = \{ \psi_1, \ldots, \psi_p \} = \text{vector of stress distribution parameters (dimension } p \} \]

The parameters of vector \( \theta_R \) can be estimated based on
previously collected reliability data for the product under
design/test or for products similar in design and applications.

Vector \( \psi_S \) characterizes the distribution of the field stress
variable and its parameters can be determined based on field
environmental studies.

The following case study will illustrate the application of this
method to an automotive reliability demonstration testing.

MODELS FOR STRESS ENVIRONMENTS

In order to illustrate this method let us consider three most
common automotive environments, which are also used in
reliability demonstration testing: high temperature exposure,
temperature cycling, and vibration. The acceleration models for
those environments can be found in a large number of
literature sources (see for example [6], Chapter 13)

High temperature acceleration is typically described by
Arrhenius model:

\[
AF_T = e^{\frac{E_A}{k \left( \frac{1}{T_{\text{field}}} - \frac{1}{T_{\text{ref}}} \right)}
\]

(7)

Where:

\( E_A \) = activation energy (typically ranging between 0.3 - 1.2 eV
for electronics)

\( k \) = the Boltzmann’s constant (8.62×10^{-5} eV/K)\)

\( T \) = the absolute temperature (Kelvin)

The most commonly used acceleration model for temperature
cycling (TC) is the Coffin-Manson, where the acceleration
factor can be represented in a simplified form as

\[
AF_{\text{TC}} = \left( \frac{\Delta T_{\text{test}}}{\Delta T_{\text{field}}} \right)^m
\]

(8)

where:

\( \Delta T_{\text{test}} \) = temperature range \( (T_{\text{Max}} - T_{\text{Min}}) \) of the TC

\( \Delta T_{\text{Field}} \) = temperature range experienced in the field

\( m \) = Coffin-Manson exponent \( (m = 2.65 \text{ is often used for Lead}
Free solder fatigue, see for example Arnold et. al. [8])

Vibration is often modeled by the S-N curve, which is similarly
to Coffin-Manson is a form of the inverse power law model.

\[
AF_{\text{Vib}} = \left( \frac{g_{\text{test}}}{g_{\text{Field}}} \right)^b
\]

(9)

Where:

\( g \) = the acceleration amplitude in the case of sinusoidal
vibration and \( g_{\text{RMS}} \) in the case of the random vibration

\( b \) = the fatigue exponent, often ranging between 4.0 and 7.0
depending on the material properties and the geometry of the
vibrated parts.

To generate the real life models and to be able to calculate field
reliability according to (6) we will use the statistical models
commonly used in the automotive reliability engineering, i.e.
two-parameter Weibull for reliability (see Appendix) and the
lognormal distribution for the stress variables, which will be
denoted here as \( f(x; \mu, \sigma) \) or \( \text{LN}(\mu, \sigma) \)

In the case where field failures follow the two-parameter
Weibull distribution (see Appendix equation (19)), field
reliability according to (3) becomes
Where $\beta$ is the Weibull slope and $\eta$ is the characteristic life.

Therefore by substituting (10) into (6) and including the acceleration models (7), (8), and (9) we will obtain respectively:

High temperature exposure:

$$R(t) = R_{Field}(t) = R_C(t) = \exp \left[ -\left( \frac{t}{\eta A F} \right)^\beta \right]$$

(10)

Temperature cycling:

$$R(t) = \int_0^{\frac{\Delta T_{ref}}{\Delta T}} \exp \left[ -\left( \frac{t}{\eta_{ref}} \right)^{\beta_{ref}} \right] f(\Delta T; \mu_{\Delta T}, \sigma_{\Delta T}) d(\Delta T)$$

(11)

Vibration:

$$R(t) = \int_0^{\frac{g_{ref}}{g}} \exp \left[ -\left( \frac{t}{\sigma_g} \right)^{\beta_g} \right] f(g; \mu_g, \sigma_g) dg$$

(12)

As mentioned before, when vibration is random the $g$ variable becomes $g_{RMS}$.

In equations (11), (12), and (13) statistical distribution parameters $\beta$, $\eta$, $\mu$, $\sigma$ have the subscripts of the three types of stress environments (temperature, TC, and vibration) and their respective tests.

**CASE STUDY: RELIABILITY DEMONSTRATION OF AN UNDER THE HOOD MOUNTED AUTOMOTIVE CONTROLLER**

Let us consider a case of a temperature cycling test simulating temperature changes caused by internal heating during on/off cycles, changes in the ambient temperature caused by diurnal cycles and by external heating coming from an engine.

12 product samples are put through a continuous thermal cycling of $[-40; 125]^\circ C$ with continuous monitoring. The test was terminated after 2000 cycles with 6 units failing during the test (see the test results in Table 1).

**Table 1. Life data for the temperature cycling test.**

<table>
<thead>
<tr>
<th>Number of test units</th>
<th>Time to Failure (cycles)</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>782</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1168</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1295</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1498</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1641</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1956</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>2000</td>
<td>Suspended after 2000 cycles (no failures)</td>
</tr>
</tbody>
</table>

In order to calculate the field reliability of this controller we need to calculate the number of cycles corresponding to one life of the product. The test is designed to simulate 10 years field life by performing continuous thermal cycling with monitoring. According to General Motors GMW3172 [1], the number of field cycles (assuming two cold start cycles per day) can be calculated as:

$$10 \text{ yr} \times 365 \text{ days/yr} \times 2 \text{ cycles/day} = 7300 \text{ cycles}.$$ 

Product specification also suggests using a high percentile user temperature value of $\Delta T_{Field} = 70^\circ C$, therefore applying equations (2) and (8) the test duration simulating one mission life will have a duration of: $7300 \left( \frac{70}{125 - (-40)} \right)^{2.65} = 752 \text{ cycles}$

The life data presented in Table 1 have been analyzed with ReliaSoft® Weibull++ v.8 using two analytical options: the rank regression (RRX) and the maximum likelihood estimator (MLE).

RRX option (50% confidence) produced: $\beta = 2.751$, $\eta = 2229.6$ cycles, therefore demonstrating the reliability at one field life $R(752 \text{ cycles}) = 0.951$

MLE (50% confidence) produced: $\beta = 2.949$, $\eta = 2247.7$ cycles with the demonstrated reliability $R(752 \text{ cycles}) = 0.961$

For the calculations of $AF_{\Delta T}$, the TC acceleration factor $\Delta T_{Test}$ in (8) is known and determined by the test specifications, on the other hand the exact value of $\Delta T_{Field} = T_{Max} - T_{Min}$ (designated further as $\Delta T$) is unknown and needs to be modeled as a random variable. Its value is subject to variation based on region’s climate and the individual drive times ($\Delta T$ is positively correlated with driving times due to continuous internal heating). It would range from low $\Delta T$ of short driving times in mild climates to high $\Delta T$ of extended driving in extreme climates.
It has been observed that statistical distributions of field usage variables, such as mileage, temperature, vibration, etc. are often skewed to the right and typically can be modeled by the lognormal distribution.

Internal Delphi field studies show that $\Delta T_{field} = 70°C$ used for the Coffin-Manson calculations corresponds to the 99th percentile severity stress level ($\Delta T_{99} = 70°C$). Field studies for these types of automotive electronics environment also show the 50th percentile temperature at $\Delta T_{50} = 40°C$ with the maximum value of $\Delta T_{max} = 100°C$. The lognormal distribution fitting this percentile data points would be $LN(3.69, 0.24)$ truncated at 100.

Therefore, substituting all the numerical values into (12) using the $\beta_{\Delta T}$ and $\eta_{\Delta T}$ obtained as a result of Weibull analysis of the life data in Table 1 and assuming lognormal distribution $f(\Delta T, \mu, \sigma) = LN(3.69, 0.24)$ produces the following numerical solutions with MATHCAD® [10]:

RRX option with the demonstrated reliability $R(752$ cycles) = 0.951

$$R(7300\text{cycles}) = \int_0^{100} \exp \left[ -\left( \frac{7300}{2292.6^{\frac{2.5}{\Delta T}}} \right)^{2.751} \right] f(\Delta T; 3.69, 0.24)d(\Delta T) = 0.9964$$

MLE option with the demonstrated reliability $R(752$ cycles) = 0.961 produces:

$$R(7300\text{cycles}) = \int_0^{100} \exp \left[ -\left( \frac{7300}{2247.7^{\frac{2.66}{\Delta T}}} \right)^{2.849} \right] f(\Delta T; 3.69, 0.24)d(\Delta T) = 0.9974$$

As you can see, the field life reliabilities $R(t_{Life})$ in both cases are significantly higher than those demonstrated by test. Rank regression: 99.64% > 95.10% and the maximum likelihood 99.74% > 96.1%. In both cases this method has shifted the ‘true’ demonstrated reliability of the product population into a more desirable region above 99%.

The mathematical approach presented in this case study would work equally well with multivariable acceleration functions like Eyring equation (e.g. Ohring [11]), Lawson Temperature Humidity model (e.g. O’Connor and Kleyner [6]), Voltage-Temperature model (e.g. SEMATECH [12]), Norris-Landzberg [13] and others. However the complexity of the calculations (4) will understandably be higher than that shown in this case study.

APPLICATION TO SUCCESS BASED TESTING

In the cases where reliability demonstration is conducted via success run testing (test to a bogey with no failures expected), equations (11) (12) (13) could still be applied, although with certain engineering assumptions. Now, let us presume that in our case study we tested 12 samples and they all passed 752 cycles (one mission life equivalent) with no failures. According to (17) (see Appendix) it would demonstrate $R(t_{Life}) = 94.34%$ reliability with $C=50\%$ confidence.

In this case the values of $\beta_{\Delta T}$ and $\eta_{\Delta T}$ for (12) could be estimated by substituting the two-parameter Weibull equation (19) into (17) (see Appendix) and making an engineering assumption. Since there is only one equation (17), it cannot be solved for both $\beta_{\Delta T}$ and $\eta_{\Delta T}$; thus one parameter (commonly the Weibull slope) needs to be assumed. Therefore we need to solve (17) and (19) for $R$:

$$R(t_{Life}) = \exp \left[ -\left( \frac{t_{Test}}{\eta_{\Delta T}} \right)^{\beta_{\Delta T}} \right] = \left( 1 - C \right)^{\frac{1}{N}} \tag{14}$$

where:

$t_{Life}$ = mission life of the product (in our case study it is 7300 field cycles)

$t_{Test}$ = test time corresponding to one mission life in the field (test to a bogey). In our case study it is 752 thermal cycles.

Solving (14) for $\eta_{\Delta T}$

$$\eta_{\Delta T} = t_{Test} \left( \frac{1}{N} \ln(1-C) \right)^{\frac{1}{\beta_{\Delta T}}} \tag{15}$$

and assuming $\beta_{\Delta T} = 2.5$ (a commonly used value in the automotive electronics, which is also consistent with the Weibull slope values in our case study) equation (15) produces $\eta_{\Delta T} = 2,352.6$ cycles.

Therefore, substituting those values into (12) will yield

$$R(7300\text{cycles}) = \int_0^{100} \exp \left[ -\left( \frac{7300}{2352.6^{\frac{2.66}{\Delta T}}} \right)^{2.5} \right] f(\Delta T; 3.69, 0.24)d(\Delta T) = 0.9954$$

and thus showing the ‘true’ field reliability of 99.54% versus the demonstrated reliability of 94.43%.
It is difficult to verify those reliability numbers with high degree of accuracy for the field life of 10 years, since vehicle warranties are typically shorter - most commonly 3 years with a few exceptions. After a warranty expiration most of the product failures are not tracked, however the analysis of Weibull curves for the design related warranty claims on a variety of powertrain controllers and the analysis of units returned to the remanufacturing centers estimates the percent of design related failures for the field life of Delphi electronic products is under 1%, which is consistent with the results in this case study.

CONCLUSION

Variations in field stress conditions may significantly affect the value of population reliability demonstrated by test. When a reliability demonstration test (either accelerated or conducted at a field stress level) is designed based on stress conditions which are more severe than the average, the actual field (‘true’) reliability will be higher than that demonstrated by test. The higher the percentile stress severity conditions for the test, the higher is the difference between the actual and demonstrated reliabilities. Since field stress conditions are not homogeneous, the calculation of the ‘true’ demonstrated reliability requires information about the relevant acceleration model(s) and the probability distributions of the stress variables over the product population in the field.

The method presented in this paper can be equally applied to both types of accelerated testing - increased stress level and increased usage rate. It is recommended to use this approach at the validation test planning stage to assure satisfaction of customer reliability requirements. This method can also be applied in reverse in order to determine the test stress level needed to demonstrate the required reliability in the cases where the sample size and test duration have already been predetermined due to resource allocations, customer requirements or cost/time restrictions.

ACKNOWLEDGEMENTS

The author would like to thank Vasiliy Krivtsov (Ford Motor Company), Alex Karagrigoriou (University of Cyprus) and Eric Juliot (Delphi Electronics & Safety) for reviewing the draft and making constructive suggestions.

REFERENCES


CONTACT INFORMATION

Andre Kleyner can be contacted at andre.v.kleyner@delphi.com.
APPENDIX

APPENDIX: RELIABILITY DEMONSTRATION METHODS

Success Based Testing
Success based (also referred as success run) testing is a technique, where survival of a product at the end of a test is expected. Under those conditions a product is subjected to a test (often accelerated), representing an equivalent of one mission life (test to a bogey), which is expected to be completed without failure by all the units in the test sample.

Success based test statistical analysis is usually based on the binomial distribution shown by (16) (see for example O'Connor and Kleyner [6])

\[ C = 1 - \sum_{i=0}^{k} \frac{N!}{i!(N-i)!} R^{N-i} (1-R)^i \]

(16)

where:

\[ R = \text{reliability} \]
\[ C = \text{confidence level} \]
\[ N = \text{total number of test samples} \]
\[ k = \text{number of failures experienced during the test}. \]

In a case with no failures \((k = 0)\) equation (16) turns into a simple form:

\[ C = 1 - R^N \]

(17)

which is commonly used in the industry to calculate the sample size for success based reliability demonstration testing.

Equation (17) can be solved for the sample size as

\[ N = \frac{\ln(1-C)}{\ln(R)} \]

(18)

For example, an automotive requirement (see [1]) may call for \(R=97\%\) with \(C=50\\%\), which would require 23 test samples which is often a costly proposition. Equation (18) also shows very clearly that with \(R\) approaching 1.0 the sample size \(N\) is approaching infinity.

Test to Failure
Reliability demonstration through test to failure is usually done by testing a product sample until at least several failures are observed and then fitting a statistical distribution (commonly the two- or three-parameter Weibull) into the collected life data. The two-parameter Weibull reliability is given by:

\[ R(t) = \exp\left[ -\left( \frac{t}{\eta} \right)^\beta \right] \]

(19)

where:

\[ \beta = \text{Weibull slope (shape parameter)} \]
\[ \eta = \text{Characteristic Life (scale parameter)} \]
More on test-to-failure and life data analysis and reliability demonstration can be found in ReliaSoft [14], Lipson and Sheth [15] and a multitude of other sources. Test to failure provides more flexibility and information regarding the product's design margins, but usually takes longer (sometimes 2× to 6× more) than success-based testing, thus extending the product design cycle and increasing the overall development cost. For example, in our case study the success based test would have taken 752 cycles, whereas our test to failure ran for 2000 cycles, while both tests demonstrated comparable reliabilities close to 95%.